CHAOS CONTROL OF CHAOTIC CHEMICAL SYSTEMS

CONTROLUL SISTEMELOR CHIMICE HAOTICE

***OANCEA Servilia*[1], *OANCEA A.V.*[2], *GROSU I.*[3]**
e-mail: liaoancea@yahoo.com

***Abstract**. Chemical systems can exhibit chaotic behaviour and this fact is very important for chemical processes and for biological structures. From this point of view the control of these phenomena have a great practical impact despite the fact that it is very dificult; this is the reason the theoretical models are useful in these situations. The control using these models can give the informations about the selfcontrol inside the biological structures where the behaviour of the dynamic systems is realized by a feedback mechanism. The main aim of this paper is to study the synchronization of two chemical chaotic systems proposed by Samardzija and which is based on 9 rections and 3 intermediary species, using the adaptive feedback method of control. The transient time until synchronization depends on initial conditions of two systems, the strength and the number of the controllers.*
***Key words**: chaotic chemical system, chaos control*

***Rezumat**. Sistemele chimice pot avea comportare haotică şi acest fat este foarte important pentru procesele chimice şi structurile biologice. Din acest punct de vedere controlul acestor fenomene are un mare impact in ciuda faptului că este foarte dificilş acesta este motivul pentru care modelele teoretice sunt utile în aceste situaţii. Contolul sistemelor pe baza acestor modele poate da informaţii despre autocontrolul din structurile biologice unde comportarea sistemelor dinamice se realizează printr-un mecanism de feedback. Scopul principal al acestei lucrări este de a studia sincronizarea a două sisteme chimice propuse de Samardzija care se bazează pe 9 reacţii şi 3 specii intermediare, folosind o metodă de control de tip feedback. Timpul după care se obţine sincronizarea depinde de condiţiile initiale ale celor doua sisteme si de intensitatea controler-ului.*
***Cuvinte cheie:** sistem chimic haotic, controlul haosului*

INTRODUCTION

Chemical reaction systems have become one of the favorite domains to study nonlinear systems, both experimentally and theoretically. These systems can exhibit chaotic behaviour and this fact is very important for chemical processes and for biological structures. From this point of view the deliberate control of these phenomena have a great practical impact despite the fact that it is very dificult; this is the reason the theoretical models are useful in these situations. In

[1] University of Agricultural Sciences and Veterinary Medicine of Iasi, Romania
[2] Erasmus Mundus Advanced Spectroscopy in Chemistry Master, Bologna University, Italy
[3] “Al. I. Cuza” University of Iasi, Romania

addition, the control using these models can give the informations about the selfcontrol inside the biological structures where the behaviour of the dynamic systems is realized by a feedback mechanism. Over the last decade, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillators especially from technical reasons. When the complete synchronization is achieved, the states of both systems become practically identical, while their dynamics in time remains chaotic. Many examples of synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies (Grosu, 1997; Grosu et al., 2008; Lerescu et al., 2004; Lerescu et al., 2006; Oancea et al., 2009; Oancea et al., 2011). The main aim of this paper is to study the synchronization of two chemical chaotic systems based on the adaptive feedback method of control. One of these chemical models was proposed by Samardzija and it is based on 9 rections and 3 intermediary species.

THEORY

The model proposed by Samardzija represents some chemical reactions and its mechanism consists in the following elementary steps (Wang and Chen, 2010):

1. $A_1 + X + 2Z \xleftrightarrow{k_1} X + 3Z$
2. $X + Y \xleftrightarrow{k_2} 2Y$
3. $A_2 + X + Z \xleftrightarrow{k_3} Z + P_1$
4. $A_3 + X + Z \xleftrightarrow{k_4} X + P_2$
5. $A_4 + 2Y \xleftrightarrow{k_5} 3Y$
6. $2Z \xleftrightarrow{k_6} P_3$
7. $A_5 + X \xleftrightarrow{k_7} 2X$
8. $Y \xleftrightarrow{k_8} P_4$
9. $A_6 + Z \xleftrightarrow{k_9} 2Z$

The time evolution of the intermediary species X, Y , and Z is given by a nonlinear system of equations:

$$\frac{dx_1}{dt} = -k_2 x_1 x_2 - k_3 x_1 x_3 + k_7 x_1$$

$$\frac{dx_2}{dt} = k_2 x_1 x_2 + k_5 x_2{}^2 - k_8 x_2$$

$$\frac{dx_3}{dt} = k_1 x_1 x_3{}^2 - k_4 x_1 x_3 - 2k_6 x_3{}^2 + k_9 x_3 \quad (1)$$

This system has a chaotic behaviour, for the following constants:

k_1 = 1 k_2 =2 k_3 =1.5 k_4 =20 k_5 =0.8

k_6=12.85 k_7 = 45 k_9=514.2

1. Chaotic dynamics of chemical system

Coosing k_8 as a control parameter, we can know dynamics of this system. For k_8 =50 the strange attractor for this system is given in the figure 1.

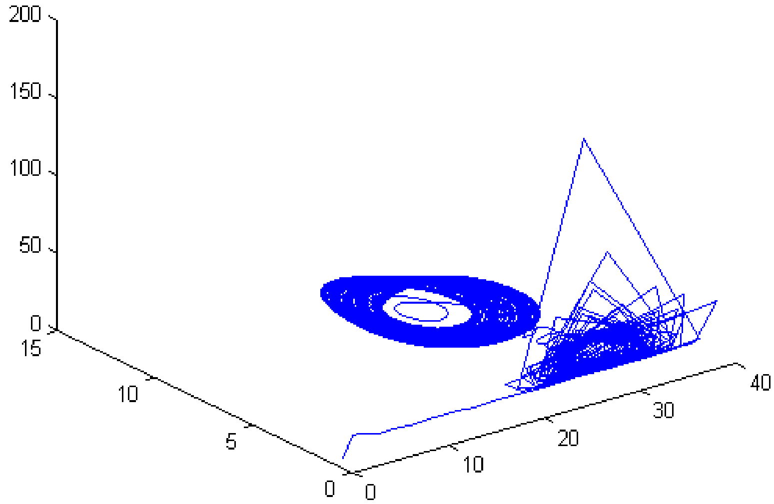
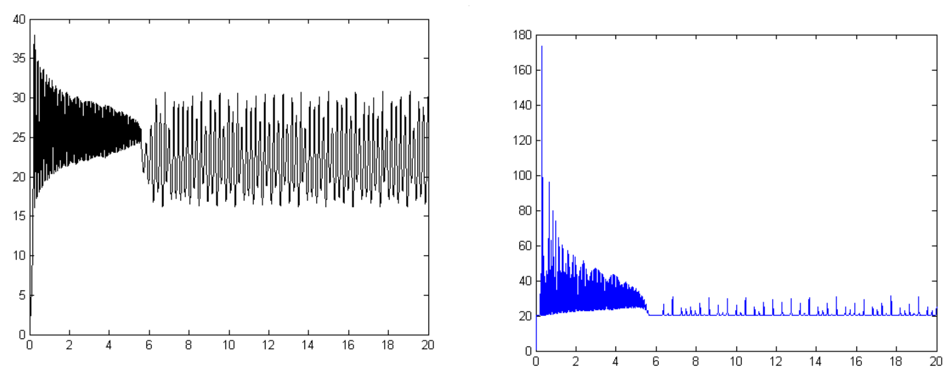


Fig. 1 – Phase portrait of (x_3, x_1, x_2,) for system (1) with initial conditions 1 1 1

The dynamics of the this chaotic chemical system is given in figure 2.



a

Fig. 2 – a-x_1(t); b- x_3(t) for k_8 =50;

The chaotic behavior is sustained by Lyapunov exponents from figure 3.

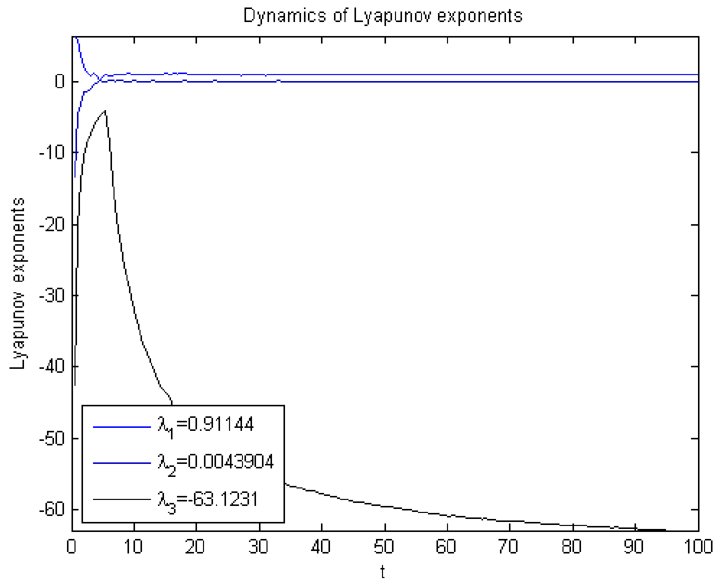


Fig. 3 – The Lyapunov exponents

This system is very sensitive to initial condition. Then we choose k_8 =43

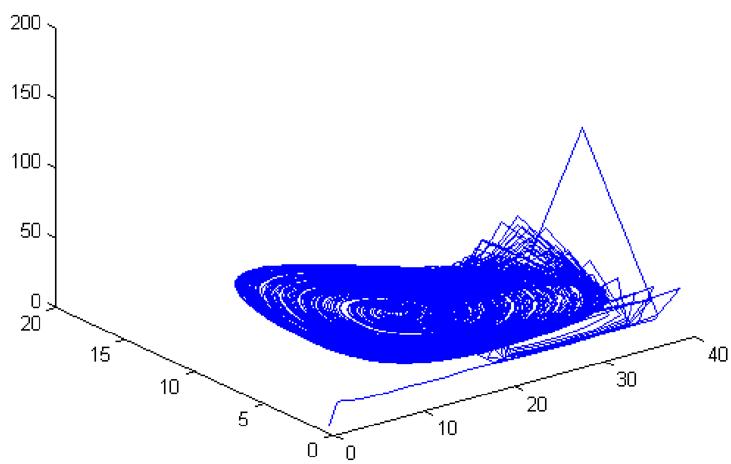
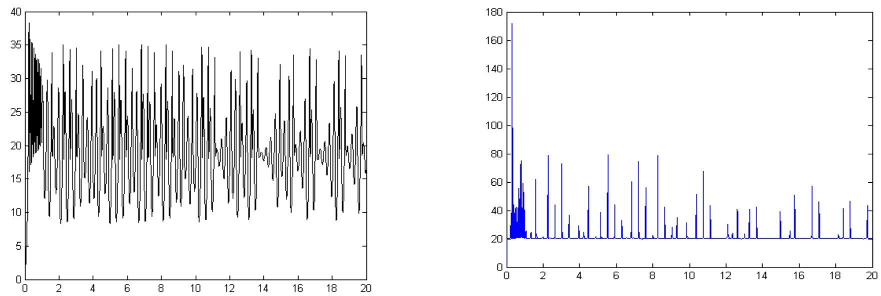


Fig. 4 – 3D attractor (x_3, x_1, x_2) for k_8 =43

Figure 5 shows the changes of the variable x and z with time for k_8 =43.



a b

Fig. 5 – a-$x_1(t)$; b- $x_3(t)$ for k_8 =43;

2. Synchronization of two chaotic systems

To synchronize two identical chemical syststems we followed the method proposed by Guo et al. [8], Hu and Xu [9], based on Lyapunov-Lasalle theory. Let be a chaotic non-autonomous system:

$$\dot{x} = f(x,t) \quad \text{where } x = (x_1, x_2 ...)^T \in R^n$$

is the state vector of the system and $f = (f_1, f_2 ..)^T \in R^n$ is the non-linear vector field of the system, which is considered as a driving system.

For any $x = (x_1, x_2 ...)^T \in R^n$ and $y = (y_1, y_2 ...)^T \in R^n$

there exists a positive constant l such that:

$$\left| f(x,t) - f(y,t) \right| \le l \max \left| x_i - x_j \right| \quad \text{i, j=1,2…n}$$

The slave system will be: $\dot{y} = f(y,t) + z(z_1, z_2, ...)$ where $z(z_1, z_2, ...)$ is the controller. If the error vector is e = y - x, the objective of synchronization is to make

$$\lim_{t \to +\infty} \left\| e(t) \right\| \to 0$$

The controller is of the form: $z_i = \varepsilon_i (y_i - x_i)$ and $\dot{\varepsilon}_i = -\gamma_i e_i^2$, i=1,2…n

and γ_i , i = 1,2,. . . , n are arbitrary positive constants.

RESULTS AND DISCUSSION

According this method of synchronization, the slave system for this chemical system will be:

$$\frac{dy_1}{dt} = -2y_1y_2 - 1.5y_1y_3 + 45y_1 + z_1(y_1 - x_1)$$

$$\frac{dy_2}{dt} = 2y_1y_2 + 0.8{y_2}^2 - k_8y_2 + z_2(y_2 - x_2)$$

$$\frac{dy_3}{dt} = y_1{y_3}^2 - 20y_1y_3 - 25.7{y_3}^2 + 514.2y_3 + z_3(y_3 - x_3) \tag{2}$$

and for the control strength:

$$\dot{z}_1 = -(y_1 - x_1)^2$$
$$\dot{z}_2 = -(y_2 - x_2)^2$$
$$\dot{z}_3 = -(y_3 - x_3)^2 \tag{3}$$

Figures 6-9 demonstrate the syncronization of the two chemical systems.

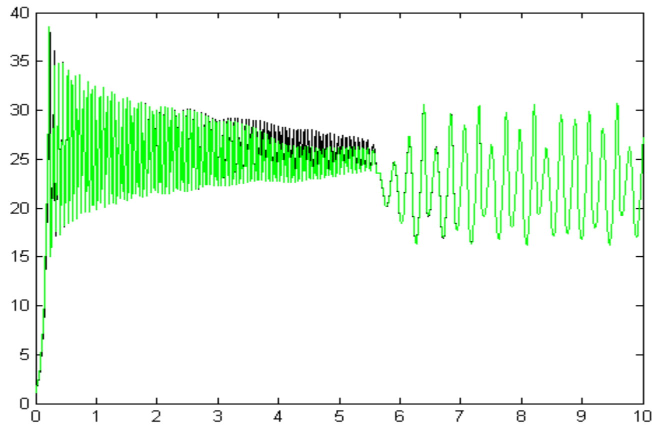


Fig. 6 – x1(t)- black y1(t)- green[$x_1(0)$=1, $x_2(0)$=1, $x_3(0)$=1; $y_1(0)$ =1.1; $y_2(0)$ = 1.1 $y_3(0)$ = 1.1; $z_1(0)$ =1; $z_2(0)$ = 1 ; $z_3(0)$ =1]

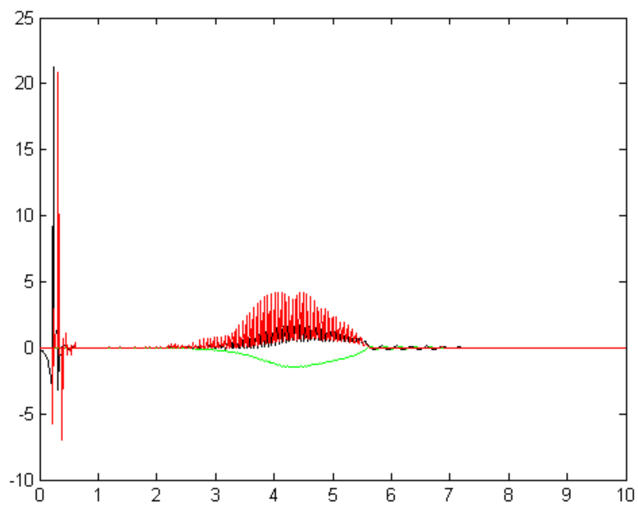
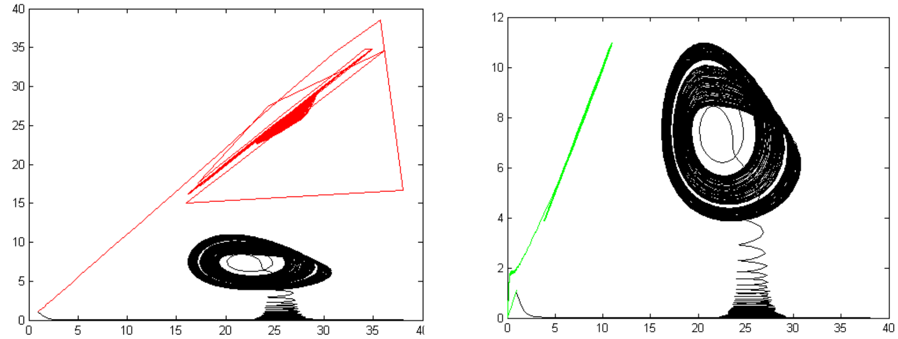


Fig. 7 – Synchronization errors between master and slave systems [$x_1(0)$=1, $x_2(0)$=1, $x_3(0)$=1; $y_1(0)$ =1.1; $y_2(0)$ = 1.1 $y_3(0)$ = 1.1; $z_1(0)$ =1; $z_2(0)$ = 1 ; $z_3(0)$ =1]



a b

Fig. 8 – Phase portrait of a) :(x_1, x_2-black) and (x_1,y_1-red); b): (x_1, x_2-black) and (x_2,y_2-green) for two systems [$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.1$; $y_2(0) = 1.1$ $y_3(0) = 1.1$; $z_1(0)=1$; $z_2(0) = 1$; $z_3(0)=1$]

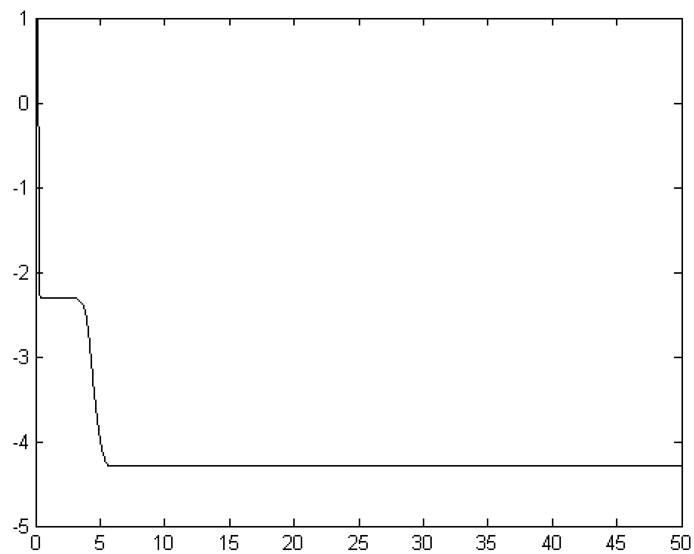
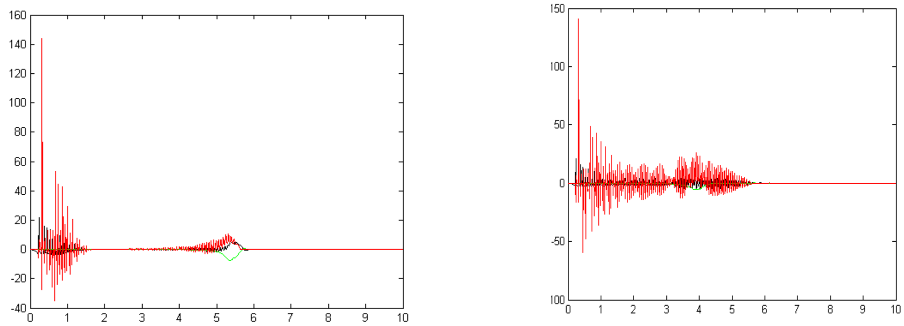


Fig. 8 – The control strength z_1[$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.1$; $y_2(0) = 1.1$ $y_3(0) = 1.1$; $z_1(0)=1$; $z_2(0) = 1$; $z_3(0)=1$]

Debin Huang (2005), by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows found that we can use a single controller to achieve identical synchronization of a three-dimensional system (for Lorenz system this is possible only we add the controller in the second equation).

For these systems we achieved the synchronization if one controller is applied only in the first or in the second equation (fig. 9).



a b

Fig. 9 – Synchronization errors between master and slave for chemical systems with one controller [$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.1$; $y_2(0) = 1.1$ $y_3(0) = 1.1$; a- $z_1(0)=1$; b- [$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.5$; $y_2(0) = 1.5$ $y_3(0) = 1.5$; $z_2(0)=1$]

CONCLUSIONS

In this work we analized the dynamics of the Samardzija system wich is based on 9 rections and 3 intermediary species and we realized the synchronization of two systems using an adaptive feedback method. The transient time until synchronization depends on initial conditions of two systems, the strength of the controllers and their number. Then we can control this chemical system in accordance with recent debates of Wang and Chen (2010) about full global synchronization and partial synchronization in a system of two or three coupled chemical chaotic oscillators.

REFERENCES

1. Grosu I., 1997 -*Robust Synchronization*, Phys. Rev. 56, pp. 3709-3712

2. Grosu I., Padmanaban E., Roy P. K., Dana S. K., 2008 - *Designing Coupling for Synchronization and Amplification of Chaos*, Phys Rev Lett 100, 234102, pp. 1-4

3. Lerescu A.I., Constandache N., Oancea S., Grosu I., 2004 - *Collection of master-slave synchronized chaotic systems*, Chaos Soliton Fract., 22(3), pp. 599-604

4. Lerescu A.I., Oancea S., Grosu I., 2006 - *Collection of Mutually Synchronized Chaotic Systems,* Physics Letters A, 352, pp. 222-228.

5. Oancea S., Grosu F., Lazar A., Grosu I., 2009 - *Master–slave synchronization of Lorenz systems using a single controller*, Chaos, Solitons and Fractals, 41, pp. 2575-2580

6. Oancea S., Grosu I., Oancea A.V., 2011 - *Biological control based on the synchronization of Lotka-Volterra systems with four competitive species*, Rom. J. Biophys, 21(1), pp. 17-26

7. Wang J.W., Chen A.M., 2010 - *Partial synchronization in coupled chemical chaotic oscillators*, Journal of Computational and Applied Mathematics, 233, pp. 1897-1904

8. Guo W., Chen S., Zhou H., 2009 - *A simple adaptive-feedback controller for chaos Synchronization*, Chaos, Solitons and Fractals, 39, pp. 316–321

9. Hu M., Yang Y., Xu Z., Guo L., 2008 - *Hybrid projective synchronization in a chaotic complex nonlinear system*, Mathematics and Computer in Simulation, 79, pp. 449-457

10. Huang D., 2005 - *Simple adaptive-feedback controller for identical chaos synchronization*, Phys. Rev. E, 71, 037203